Appendix
A Short History of 'Definition'

Abstract: A short history of 'definition' in recent philosophy is presented. The views presented here are in approximate chronological historical order and revolve around discussion of 'real' and 'nominal' definitions. Under the tripartite theory, a 'real' definition is approximately identified as a 'theoretic' definition. A 'nominal' definition is understood to be identified as 'reportive' or 'stipulative.' The conclusion of this history is that the quoted texts support the idea that mathematical definitions are typically (3b) stipulative abbreviations or (3c) stipulative formalizations of fixed definiens concepts.

The concept of 'definition' lacks a strong history of evolving thought. Many discussions are centered around the distinction between 'real' and 'nominal' definitions. In this essay, views of Aristotle, Leibniz, Mill, Frege, Schlick, Tarski, Russell, Suppes, Popper, Ayer, Robinson, Quine, Cohen & Nagel, and Abelson are presented in approximate chronological order. In more contemporary discussions about 'definition' the philosophical talk tends to fluctuate between 'natural kind' definitions about physical entities (e.g., Kornblith, 1993) and 'stipulative' definitions that are most applicable to the practice of the deductive sciences (e.g., Suppes, 1957). The notions of 'substitutivity' and 'synonymy' are also common themes and important when discussing what definitions are.

The views of Aristotle and Plato are both extremely influential in framing the discussion of 'definition.' Aristotle's theory of definition involved resolving distinctions between the *genus* and *differentia* of entities.[1] In contrast, Plato thought that the definiens of a definition, was the analysis of the form, idea, or universal symbolized by a term. Besides Aristotle's and Plato's views, and short analyses in essays and books, there has been little extended theoretic work done on the concept of 'definition' in modern times. Before Quine's "Natural Kinds" (1969), most modern talk about definition had come from logicians and mathematicians. In philosophy, physical scientific 'natural kind' theoretic definitions are now also widely discussed.

[1] In a medieval text, Boethius (480-526) praises Aristotle's work on definition in the *Posterior Analytics* (II 10, 93b29-94a19) suggesting that there are two implied rules of definition. One of these rules about the art of definition is that "Of things, some are higher, others lower, and others intermediate. Certainly, no definition encompasses higher things, for no genera higher than they can be found. On the other hand, however, the lower things, such as individuals, lack specifying differentiae; for that reason, they too are excluded from definitions. Therefore, it is the intermediate things, those that have genera and that are predicated of the others- of genera, of species, or of individuals- that can fall under definition" (Kretzmann and Stump, 1988, pp. 28-29).

Aristotle and Mill

In his translation and commentary of *Euclid: The Thirteen Books of the Elements (Vol. 1)* (1925, 1956), Sir Thomas Heath discusses the history of 'definition' with references to Aristotle and J.S. Mill:

> It is necessary, says Aristotle, whenever anyone treats of any whole subject, to divide the genus into its primary constituents, those which are indivisible in species respectively: e.g. number must be divided into triad and dyad; then an attempt must be made in this way to obtain other definitions, e.g. of a straight line, of a circle, and of a right angle.
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> The (Greek) word for definition is x. The original meaning of the word seems to have been 'boundary, 'landmark'…
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> Let us first be clear as to what a definition does *not* do. There is nothing in connexion with definitions which Aristotle takes more pains to emphasize than that a definition asserts nothing as to the *existence* or *non-existence* of the thing defined. It is an answer to *what* a thing is, and does not say *that* it is. The existence of various things defined has to be proved, except in the case of a few primary things in each science, the existence of which is indemonstrable and must be *assumed* among the first principles of each science; e.g. points and lines in geometry must be assumed to exist, but the existence of everything else must be *proved*. This is stated clearly in the long passages quoted under 'First Principles' (*Anal post.* II. 13, 96b15) …
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> (This underscores the) … very fact which Mill pointed out in his discussion of earlier views of Definition, where he says that the so-called *real* definitions or definition of *things* do not constitute a different kind of definition from *nominal* definitions, or definitions of *names*; the former is simply the latter *plus* something else, namely a covert assertion that the thing defined exists. "This covert assertion is not a definition but is a postulate. The definition is a mere identical proposition which gives information only about the use of language and from which no conclusion affecting matters of fact can possibly be drawn. The accompanying postulate, on the other hand, affirms a fact which may lead to consequences of every degree of importance. It affirms the actual or possible existence of Things possessing the combination of attributes set forth in the definition; and this, if true, may be foundation sufficient on which to build a whole fabric of scientific truth" …[2]
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> In modern times, too, Mill's account of the true distinction between *real* and *nominal* definitions had been fully anticipated by… (others)… (and the *nominal* and *real* distinction is where)… the former are only intended to explain the meaning that is to be attached to a given term, whereas the latter, beside declaring the meaning of a word, affirm at the same time the existence of thing defined or, in geometry, the possibility of constructing it…(Heath, pp. 143-145).

[2] From J.S. Mill, *System of Logic*, Bk I, Ch viii. Heath goes on to criticize Mill's remarks as adding nothing to Aristotle's doctrine.

Heath briefly describes how nominal definitions may be transformed into real definitions by means of demonstration, and defends Euclid, saying that Euclid was not guilty of error of presupposing the existence of a square, because he supposed it only after constructing it at *The Elements* I.46. Heath goes on to detail Aristotle's requirements for definition.

Leibniz

Gottfried Wilhelm Leibniz (1646-1716) understood definitions in line with Aristotle's *genus* and *differentia* conception as stated above. G. MacDonald Ross (1984) describes Leibniz's view of 'definition' as follows:

> One of the more conventional aspects of Leibniz's logic was his acceptance of the tradition of definition by genus and differentia (the 'method of division'). According to this approach, which ultimately stemmed from Plato and Aristotle, the proper method of defining classes of things was to start with a very general class (the *genus*) and divide it into two smaller, mutually exclusive classes (*species*) by means of some property which every member of the genus either did or did not have (the *differentia*).
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> Whole trees of branching classes could be defined in this way. One of the simplest and best-known examples was the so-called 'Tree of Porphyry,' named after the third-century commentator on Aristotle's *Categories*. This started by dividing things into the material (bodies) and the immaterial; bodies into the animate (living things) and the inanimate; living things into those who had sensation (animals) and those that did not (vegetables); and animals into the rational (man) and the non-rational (brutes)…
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> In common with many of his contemporaries, Leibniz believed that, in principle, all concepts could be defined in terms of their position in one single hierarchy of this sort. And however unrealistic the ideal, it had important repercussions on his philosophy as a whole.
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> The method of division encouraged a belief that, at any stage, the concept defined must be more *complex* than the concepts used to define it. So, if the concept 'man' was a combination of the concepts 'rational' and 'animal,' these components had to be simpler than the concept compounded from them. It seemed to follow that, ultimately, there must be certain absolutely simple, atomic concepts out of which all others were constructed, otherwise there would be an infinite regress. In particular, the very first, or 'highest' genus could not be defined as a species of any higher genus; and the various differentiae had to be either simple concepts themselves, or else reducible to simples by a process of definition. Obviously, any philosophy structured around the method of division had the problem of identifying the simple concepts, and of explaining how we could acquire them (pp. 51- 52).

John Stuart Mill

John Stuart Mill (1806-1873) in his *A System of Logic, Ratiocinative and Inductive* (1882) devoted chapter VIII to the essay "Of Definition." Mill's views about 'definition' are heavily influenced by his philosophy of language concerning the 'denotation' and 'connotation' of words. According to Mill, words *denote* the *objects* which they are true of, and *connote* specific *attributes* of those objects. The word 'man' *denotes*, or is true of, *all men*—"Peter, Paul, John, and an indefinite number of individuals." But it *connotes* the *attributes* in virtue of which the word 'man' applies to these individuals—"corporeity, animal life, rationality, and a certain external form, for which distinction we call human" (VII, p. 31).[3] But not all words have connotation. Mill maintains, in historical tradition, that words can be 'singular' or 'general.' *'Cicero'* is a *singular name*, applying to only one object, namely Cicero. *'Roman'* is, by contrast, a *general name*, which applies to many objects, including Cicero, Augustus, Nero, and many others. Singular proper names such as 'Cicero' don't connote any attributes, while general names such as 'Roman' have connotation (e.g. a place of residency, legal citizenship, etc.)

Mill's theory is a contribution to the 'semantics' of proper names; that is, about what proper names contribute to the meanings of sentences. What ties a proper name to its referent? For Mill, proper names, e.g. of individual persons, are merely labels or objects that contribute no more than the individuals themselves to the meaning of sentences in which they occur. A proper name's sole function is to introduce that individual into a discourse, while lending nothing else to the meaning of a sentence in which it occurs. If we say, "Fred is fat," the meaning of the sentence consists simply of the person Fred himself concatenated with the property of being fat. Proper names do not

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[3] In "Of Names" (Chapter I) Mill says "A non-connotative term is one which signifies a subject only, or an attribute only. A connotative term is one that which denotes a subject and implies an attribute. By a subject is here meant anything which possesses attributes. Thus John, or London, or England are names which signify a subject only. Whiteness, length, virtue, signify an attribute only. None of these names, therefore, are connotative. The word white denotes all white things, as snow, paper, the foam of the sea, etc., and implies, or in the language of schoolmen, *connotes*, the attribute *whiteness*. The word *white* is not predicated of the attribute, but of the subjects, snow, etc.; but when we predicate it of them, we convey the meaning that the attribute whiteness belongs to them."

indicate or imply any attributes to those individuals which they refer to (II). There is no more to the meaning of a name than the fact that it refers to the object it does refer to.[4]

When speaking explicitly about 'definition,' Mill says that:

> The simplest and most correct notion of a *Definition* is, a proposition *declaratory of the meaning of a word*; namely, *either* the meaning which it bears *in common acceptation*, *or* that which *the speaker or writer*, for the purposes of his discourse, *intends to annex to it* (VIII, p.164, italics added).

Mill in the above quote, vaguely recognizes what have been identified as 'reportive' and 'stipulative' definitions in the tripartite theory. But Mill alternatively *rejects* that 'theoretical definitions' (or 'real definitions') as identified here, count as 'definitions' because their purpose is to *classify*, but *not state the meaning* of a term. Also, scientific classifications are continually modified as scientific knowledge advances:

> "Man is a mammiferous animal having two hands" is the scientific definition of man, considered as one of the species in Cuvier's distribution of the animal kingdom. In cases of this sort, though the definition is still a declaration of the meaning which in the particular instance the name is appointed to convey, it cannot be said that to state the meaning of the word is the purpose of the definition. The purpose is not to expound a name, but a classification. The special meaning which Cuvier assigned to the word Man (quite foreign to its ordinary meaning, though involving no change to the denotation of the word), was incidental to a plan of arranging animals into classes on a certain principle, that is, according to a certain set of distinctions… Scientific definitions, whether they are definitions of scientific terms, or of common terms used in a scientific sense, are almost always of the kind last spoken of: their main purpose is to serve as landmarks of scientific classification. And since the classifications in any science are continually modified as scientific knowledge advances, the definitions in the sciences are also constantly varying. (VIII, p.172).
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> There is a real distinction, then, between definitions of names, and what are erroneously called definitions of things; but it is the latter, along with the meaning of a name, covertly asserts a matter of fact. This covert assertion is not a definition, but a postulate. The definition is a mere identical proposition, which gives information only about the use of language, and from which no conclusions affecting matters of fact can possibly be drawn (VIII, p.178).

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[4] In "Of Names" (Chapter I) Mill says "Proper names are not connotative; they denote the individuals who are called by them, but they do not indicate or imply any attributes as belonging to those individuals. When we name a child by the name Paul or a dog by the name Caesar, these names are simply marks used to enable those individuals to be made subjects of discourse… Proper names are attached to objects themselves and are not dependent on the continuance of any attribute of the object."

Based upon his theory of connotation and denotation, and the semantics of proper names, Mill declares that proper names *cannot* be defined:

> The definition of a word being the proposition which enunciates it meaning, words which have no meaning are unsusceptible of definition. Proper names, therefore, cannot be defined. A proper name being a mere mark put upon an individual, and of which it the characteristic property to be destitute of meaning, its meaning cannot of course be declared; though we may indicate by language, as we might indicate still more conveniently by pointing with the finger, upon what individual that particular mark has been, or is intended to be put… In the case of connotative names, the meaning, as has been so often observed, is the connotation' and the definition of a connotative name, is the proposition which declares its connotation. (VIII, pp.164-165).

In sum, Mill's 'theory of definition' *opposes* the 'tripartite theory' and the 'theory of speaker reference' given above, in *denying* both that theoretical definitions are a kind of definition, and that proper names are susceptible to a reportive definition. Also, contrary to the philosophy of language described above, Mill believes that 'proper names attach to objects themselves,' which seems clearly metaphorical and literally an error.

Gottlob Frege

Gottlob Frege (1848-1925) maintained that in a formal theory "a definition of a concept (of a possible predicate) must be complete; it must unambiguously determine, as regards to any object, whether or not it falls under the concept (whether or not the predicate is truly assertable of it). Thus, there must not be any object as regards which the definition leaves in doubt whether it falls under the concept... the concept must have a sharp boundary" (Geach, ed. 1970, p. 159). The definition of a 'concept' must satisfy the law of excluded middle: for every object x, either x falls under a concept, or it does not. Frege believed that concepts such as 'bald person' or 'heap' were ambiguous imperfections of ordinary language, and were to be avoided in a logical language, because the extensions of what persons are to count as bald, and the number of beans needed to constitute a heap can't be clearly demarcated. Vague predicates are to be banned from a logical language. With a precise formal language, the definition and meaning of a term should be specified by a set of conditions that are necessary and sufficient for the correct application of that term. In *Begriffshrift* (1879), a key piece of his formal work, every symbol introduced was given a fixed sense and fixed denotation (or extension).

Frege believed that the definiens of a definition should use terms that are already familiar, but that the (stipulated) sense of the definiens should show its worth by proving fruitful in allowing a proof of thoughts not previously demonstrated. Frege says a new name is introduced by means of a *definition* by stipulating that it is to have the same sense and the same denotation as some name composed of signs that are familiar. A definition assigns sense and reference to the symbol it introduces by postulating that the definiens is to have the same sense and the same denotation as the definiendum. Definitions abbreviate a definiendum to an explicitly stated definiens, but any definition is theoretically eliminable by substituting the longer linguistic definiens for the definiendum. Definitions are assumed stipulations that are neither true nor false.[5]

Moritz Schlick

Moritz Schlick (1882-1936), a logical empiricist, explicitly attempted to account for how 'concepts' and 'definitions' are related to each other. Curiously, Schlick (1925) believed that a concept is "not a real mental structure of any sort":

> What is a concept? A concept is to be distinguished from an intuitive image… by the fact that it is completely determined and has nothing uncertain about it. One might be tempted to say—and many logicians have indeed said—that a concept is simply an image with strictly fixed content. As we have seen, however, there are no such entities in psychological reality because all images are to one degree or another vague. One might of course suppose that images with fixed content are at least possible; but this supposition would still be limited to individual images. It would not apply at all in the case of general ideas or images, and these are what we need for knowing… general images cannot possibly exist as real mental entities.
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> Thus, a concept is not an image. It is not a real mental structure of any sort. Indeed, it is not real at all, but imaginary—something we assume in place of images with strictly determined content. We operate with concepts as if they were images with exactly delineated properties that can always be re-cognized with absolute certainty. These properties are called the *characteristics* or *features* (*Merkmale*) of the concept and are laid down by means of specific stipulation

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[5] Frege acknowledged that there was a tension between the view that definitions are abbreviations, thus being trivial equivalences of linguistic expressions, and that definitions should have a systematic fruitfulness. His final thoughts in 1914 moved toward a view that definitions should work to be a precise reconstruction of what is already implicit in the ordinary use of a concept (e.g. 'number'). A detailed description of Frege's view of definition is found in Resnik (1980), Dummett (1981), and Horty (2007).

> which in their totality constitute the *definition* of the concept. In logic, the totality of the characteristics of a concept is called its "intension" (or "content"); the set of objects denoted by the concept is called its "extension."
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> Thus, it is through definitions that we seek to obtain what we never find in the world of images but must have for scientific knowledge: absolute constancy and determinateness. No longer is the object to be known compared with vague images; instead we investigate whether the object possesses certain properties fixed by definition. In this way, it becomes possible to know the object, that is to designate it by its right name. For the definition specifies the common name we are to apply to all objects that possess the characteristics set forth in the definition. Or, to use the traditional language of logic, every definition is a nominal definition (p. 20).

The details of Schlick's theory are not important here. Schlick admits that his view that concepts do not actually exist, and his auxiliary theory that all that really exist are *conceptual functions*, "encountered widespread opposition" (p. 24). Schlick did, however, appreciate the importance of David Hilbert's theory of implicit definition:

> David Hilbert undertook to construct geometry on the foundation whose absolute certainty would not be place in jeopardy at any point by an appeal to intuition… The task was to introduce the basic concepts, which are in the usual sense indefinable, in such a fashion that the validity of the axioms that treat of these concepts is strictly guaranteed. And Hilbert's solution was simply to stipulate that the basic or primitive concepts are to be *defined* just by the fact that they satisfy the axioms.
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> This is what is known as definition by axioms, or definition by postulates, or implicit definition…
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> Modern mathematics, in electing to define the basic concepts of geometry in this manner, is not really creating something new and exceptional. It is merely uncovering the role that these concepts actually play and have always played in mathematical deduction. That is to say, when we deduce mathematical truths from one another, the *intuitive* meaning of the basic concepts is of no consequence whatsoever. In so far as the validity and interconnection of mathematical propositions are concerned, it makes no difference whether, for example, we understand by the word 'plane' the familiar intuitive figure everyone thinks of when he hears that word, or any other figure. What matters is only that the word means something for which a particular set of statements (the axioms) holds. And exactly the same thing is true of the remaining concepts that occur in these axioms. They too are defined solely by the fact that they stand in relation to other concepts (pp. 33-34)

Schlick compares "the meaning and effect" of *implicit definitions* and how they differ from *ordinary (nominal) definitions*:

> In the case of ordinary definitions, the defining process terminates when the ultimate indefinable concepts are in some way exhibited in intuition (concrete definition). This involves pointing to something that is real, something that has individual existence. Thus, we explain the concept of point by indicating a grain of sand, the concept of straight line by a taut string, that of fairness by pointing to certain feelings the person being instructed finds present in the reality of his own consciousness. In short it is through concrete definitions that we set up the connection between concepts and reality. Concrete definitions exhibit in in intuitive or experienced reality that which henceforth is to be designated by a concept. On the other hand, implicit definitions have no association or connection with reality at all; specifically, and in principle they reject such association; they remain in the domain of concepts. A system of truths created with the aid of implicit definitions does not at any time rest on the ground of reality. On the contrary, it floats freely, so to speak, and like the solar system bears within itself the guarantee of its own stability. None of the concepts that occur in the theory designate anything real; rather, they designate one another in such fashion that the meaning of one concept consists in a particular constellation of a number of the remaining concepts. (p. 37)

Alfred Tarski

Alfred Tarski (1901-1983) who provided a semantic conception of truth, says this in *Introduction to Logic and the Methodology of Deductive Sciences* (1946):

The Formulation of definitions and its rules: The phrase *"if and only if"* is very frequently used in laying down DEFINITIONS-- that is conventions stipulating what meaning is attributed to an expression which thus far has not occurred in a certain discipline, and which may not be immediately comprehensible. Imagine, for instance, that in arithmetic the symbol '<' (underlined) has not yet been employed but one wants to introduce it now into the considerations (looking upon it, as usual, as an abbreviation of the expression *"is less than or equal to"*). For this purpose, it is necessary to define this symbol first, that is, to explain its meaning in terms of expressions which are already known, and those meanings are beyond doubt (p. 34).

If a definition is to fulfill its proper task well, certain precautionary measures have to be observed in its formulation. To this effect special rules are laid down, the so-called RULES OF DEFINITION, which specifies how definitions should be constructed correctly. Since we shall not go into an exact formulation of these rules, it may be remarked that, on their basis, every definition may assume the form of an equivalence; the first member of that equivalence, the DEFINIENDUM, should be a short grammatically simple sentential function of an arbitrary structure, containing only constants whose meaning either is immediately obvious or has been explained previously... In order to emphasize the conventional character of definition and to distinguish it from other statements which have the form of an equivalence, it is expedient to prefix it by the words such as *"we say that"* (p. 35).

Tarski believes definitions should stipulate a definiendum to an already meaningful definiens. A definiendum should be eliminable into its longer equivalent definiens, and a definiens should not be 'creative' by including terms and entities not already defined.[6] The definiens must be equivalent to the definiendum, and it be applicable to everything which of which the definiendum can be predicated and applicable to, and nothing else.[7]

Bertrand Russell & Alfred North Whitehead

Bertrand Russell (1872-1970) and Alfred North Whitehead (1861-1947) in *Principia Mathematica* (2nd edition, 1903, p. 11) endorse a view similar to Tarski's:

> A definition is a declaration that a certain newly introduced symbol or combination of symbols is to mean the same as a certain other combination of symbols of which the meaning is already known. It is to be observed that a definition is, strictly speaking, no part of the subject in which it occurs. For a definition is concerned wholly with symbols, not with what they symbolize. Moreover, it is not true or false, being an expression of volition, not of a proposition.

This view of definition, adopted by Russell, Whitehead, Tarski, and to some extent Frege, has been termed the 'standard view' or the 'official view' of mathematical definition by authors James Robert Brown (1999) and John Horty (2007). The standard view maintains that definitions are constructions that are neither true nor false. Definitions posit an abbreviation of a linguistic definiendum to a linguistic definiens. Definitions are stipulated for clarity and convenience. Most often the equal sign (=), the bi-conditional sign (iff), or a definition sign (df) are used to state that the linguistic sign on the left side

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[6] In speaking of RULES OF DEFINITION, Tarski is apparently referring to the rules of 'eliminability' and 'non-creativity.' These two criteria for proper definition are attributed to Stanislaw Lesniewski (1886-1939) and are explained by Suppes (1957) and are discussed below.

[7] In his "Tarski's Theory of Definition" (2008), Wilfrid Hodges expresses disappointment about how little Tarski had to say about definitions. Hodges had expected Tarski to have well-formed views on definitions, but Tarski's statements about definition were "often indirect, and sometimes frankly careless." In contrast, Tarski was much more careful when discussing the relationship between object theory and meta-theory.

(i.e. definiendum) is the same (or is identical) to the content on the right (i.e. the definiens). Philosophical (and mathematical) definitions are intended to be 'neutral' in content. Definitions should play no substantive role among the premises of a deductive argument and should play no role in the outcomes of deductive proofs and arguments. This stipulated 3b 'abbreviatory view' is widely accepted as standard among philosophers and logicians.[8]

Karl Popper

This standard abbreviatory concept of definition is endorsed by the noted philosopher of physical science, Karl Popper (1902-1994). He argues against Aristotelian 'essentialism' where a scientific definiens is conceived as an empirical analysis of genus and species. In "Two Kinds of Definition" (1945) he argues that scientists don't pursue this kind of Aristotelian definition in modern physical science.[9] He states that:

> In modern science, only nominalist definitions occur, that is to say, shorthand symbols or labels are introduced in order to cut a long story short. And we can at once see from this that definitions do *not* play any important part in science. For shorthand symbols can always, of course, be replaced by the longer expressions,

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[8] Besides the 'standard view,' Russell in *Principles of Mathematics* (1903, pp. 111-12) says the following about mathematical definitions. I have altered Russell's original quote by substituting 'axiom' for his synonymous term 'notion':

: > Now definability is a word which, in Mathematics, has a precise sense, though one which is relative to some given set of axioms (e.g. Peano's). Given any set of axioms, a term is definable by means of these axioms when, and only when, it is the only term having to certain of these axioms a certain relation which itself is one of the said axioms. But, philosophically, the word *definition* has not, as a rule, been employed in this sense; it has, in fact, been restricted to the analysis of an idea into its constituents. This usage is inconvenient and, I think, useless; moreover, it seems to overlook the fact that wholes are *not*, as a rule, determinate when their constituents are given, but are themselves new entities (which may be in some sense simple), defined, in the mathematical sense, by certain relationships to their constituents. I shall therefore, in future, ignore the philosophical sense, and speak only of mathematical definability.

Russell's concept of 'mathematical definition' is consistent with the concept of 'implicit definition' found in Hilbert's formalism. In the first paragraph of the same edition, Russell provides a three-sentence definiens of 'pure mathematics' as a class of propositions of the form 'p implies q.' He admits that his definiens of 'pure mathematics' is unusual, but says that it is capable of justification: "The definition professes to be, not an arbitrary decision to use a common word in an uncommon signification, but rather a precise analysis of the ideas which, more or less unconsciously, are implied in the ordinary employment of the term. Our method will therefore be one of analysis, and our problem may be called philosophical- in the sense, that is to say, that we seek to pass from the complex to the simple, from the demonstrable to its indemonstrable premises" (p. 3).

[9] "Two Kinds of Definition" is found as chapter six in *Popper Selections* (1985), edited by David Miller. It originally appeared as section II of chapter eleven in Popper's, *The Open Society and Its Enemies* (1950).

> the defining formulae, for which they stand. In some cases, this would make our scientific language very cumbersome; we should waste time and paper. But we should never lose the slightest piece of factual information. Our 'scientific knowledge,' in the sense in which the term may be properly used, remains entirely unaffected if we eliminate all definitions; the only effect is upon our language, which would lose, not precision, but merely brevity… scientific or nominalist definitions do not contain any knowledge whatever… they do nothing but introduce new arbitrary shorthand labels; they cut a long story short (pp. 92-93).

Patrick Suppes

The standard view is likewise assumed by Patrick Suppes (1922-1914) in *Introduction to Logic* (1957). In a chapter titled 'A Theory of Definition,' he discusses the 'rules of definition.' Definitions should satisfy the conditions of 'eliminability' and 'non-creativity.' These criteria for proper definitions are attributed to Stanislaw Lesniewski (1886-1939) and are implicit in the writings of Frege and Tarski. Under these rules, we start with undefined terms, called primitives; and then

> (1) we must always be able to replace any defined term in favor of primitive ones (eliminability) and
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> (2) no new theorems should be proven with help of definitions that could not be proven without them (non-creativity).

In criticism, while these two criteria for definitions might be relevant (and respected) as rules for solving deductive proofs *from an already specified set of adopted axioms*, its seems that these rules are not relevant to understanding what definitions are, in general.[10] Suppes' elaboration of the rules for mathematical definition are technical. Whether the rules of eliminability and non-creativity "govern" the practice of logicians and mathematicians when investigating systems of axioms, will not concern us here. [11]

[10] Suppes says that "In laying out a given theory for investigation we want to state the creative axioms at the beginning and always refer to them as 'the axioms.' Since the definitions are theoretically dispensable, we do not want to give them the same status as the basic axioms of the theory" (p. 154).

[11] Another example of the standard view is found in a contemporary geometry text, *The Foundations of Geometry* (2002) by Gerard Venema. Venema states that "the role of definition is just to allow statements to be made concisely. For example, we will define three points to be collinear if there is one line such that all three points lie on that line. It is much more clear and concise, to say that three points are noncollinear than it is to say that there does not exist a single line such that all three points lie on that line" (p. 18).

A.J. Ayer

A view in the tradition of 'necessary and sufficient conditions' for philosophical definitions is that of A.J. Ayer (1910-1989) in his *Language, Truth, and Logic* (1946). Ayer claims that definitions should clarify logical equivalences between linguistic expressions (used in ordinary language) with a more precise language:

> We define a symbol *in use,* not by saying that it is synonymous with some other symbol, but by showing how the sentences in which it significantly occurs can be translated into equivalent sentences, which contain neither the *definiendum* itself, nor any of its synonyms. A good illustration of this process is provided by Bertrand Russell's so-called theory of definite descriptions, which is not a theory at all in the ordinary sense, but an indication of, the way in which all phrases of the form "the so-and-so" are to be defined. It proclaims that every sentence which contains a symbolic expression of this form can be translated into a sentence which does not contain any such expression, but does contain a sub-sentence asserting that one, and only one, object possesses a certain property, or else no one object possesses a certain property. Thus, the sentence "The round square cannot exist" is equivalent to "No one thing can be both square and round"; and the sentence "The author of *Waverly* was Scotch" is equivalent to "One person, and one person only, wrote *Waverly* and that person was Scotch." (p. 60).
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> The effect of this definition of descriptive phrases, as of all good definitions, is to increase our understanding of certain *sentences* (p. 61 italics added).
>
> In general, we may say that it is the purpose of a philosophical definition to dispel those confusions which arise from our imperfect understanding of certain types of sentence in our language, where the need cannot be met by the provision of a synonym for any symbol, either because there is no synonym, or else because the available synonyms are unclear in the same fashion as the symbol to which the confusion is due (1946, p. 62).

Ayer believes that a conceptual analysis should identify the conditions for the correct application of a linguistic entity (symbol, phrase, sentence) that allows proper reference to its 'objects' or 'extensions' (if any). With the positivist tradition of the twentieth century, it was thought that interesting concepts could be clarified with the specification of the necessary and sufficient conditions for a linguistic expressions' proper use.[12]

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[12] At page 85, Ayer says that the symbolic expression "7+5=12" is synonymous with "12." This isn't true. If a farmer plants six rows of corn in columns of two, the reasoning for knowing that the total number of corn stocks is 12, is explained by the computation of '6 x 2,' not '7+5=12' nor '14-2=12.' Even though these three computational expressions denote 12, these symbolic notations are not synonymous with 12.

W.V.O. Quine

W.V.O. Quine (1908-2000) discusses the concept of definition in several of his writings. In his essay, "Two Dogmas of Empiricism" in *From a Logical Point of View* (1953) he casually observes that there are roughly three kinds of definition: (1) those that are 'lexical' in the sense of reports of instances of synonymy as reports upon usage, (2) those kinds of definitions that do not limit themselves to preexisting synonymies, what Rudolph Carnap calls 'explication,' where the purpose is not merely to paraphrase the definiendum into an outright synonym, but to actually improve upon the definiendum by refining or supplementing its meaning; the explication is to preserve the usage of favored contexts, while sharpening the usage of other contexts, and (3) a third extreme sort of definition is one which does not hark back to prior synonymies at all, where there is an explicitly conventional introduction of a novel notation for the purpose of abbreviation. With this third sort of definition, the definiendum becomes synonymous with the definiens (pp. 24-26). Quine believes that 'definition' is of less theoretical importance than that of understanding what 'synonymy' is. He devotes just a few pages to sketch a concept of 'definition,' and later discusses the role of mathematical definition in the foundations of mathematical logic (pp. 82-83).[13]

Richard Robinson

In the final chapter of his book *Definition* (1954), Richard Robinson (1902-1996) offers criticism of Russell and Whitehead's concept of definition in "Definitions in Mathematics." Robinson points out that the statement that "a definition is a declaration that a certain newly-introduced symbol or combination of symbols is to mean the same as a certain other combination of symbols of which the meaning is already known" is quite

[13] In "Truth by Convention" (1936) Quine says: "Functionally a definition is not a premise to a theory, but a license for rewriting theory by putting definiens for definiendum or vice versa. By allowing such replacements a definition transmits truth; it allows true statements to be translated into new statements which are true by the same token…Although signs introduced by definition are formally arbitrary, more than such arbitrary convention is involved in questions of definability; otherwise any expression might be said to be definable on the basis of any expressions whatever. When we speak of definability, or of finding a definition for a given sign, we have in mind some traditional usage of the sign antecedent to the definition in question. To be satisfactory in this sense a definition of the sign not only must fulfill the formal requirement of unambiguous eliminability, but must conform to traditional use in question" (pp. 330-31).

untenable. It is evident that many definitions in mathematical systems are something other than abbreviations. He considers two definitions (pp. 194-195):

> 1) The symbol 'horseshoe' is to mean the same as 'implies.'
> 2) '**p** implies **q**' is equivalent to 'either **p** is false, or **q** is true.'
>
> The first of these is an abbreviation, but the second is not. The first is useful but uninteresting; the second is very interesting and has given rise to great discussion. It is not a mere 'typographical convenience.' Its fundamental difference from the first is connected with the fact that, whereas the horseshoe (i.e. symbol backwards 'c') means nothing before its definition, the word 'implies' means something before the second definition and continues to mean it afterwards (p. 194).

Robinson states the first definition of 'implies' (the horseshoe symbol) is an *abbreviation* of a new symbol to a longer meaningful definiens. The second definition of 'implies' involves a meaningful term that is modified or made more *precise* for technical use.

Robinson's recognition of these two kinds of definition is consistent with the tripartite distinction between a definition that presents a notational *abbreviation* of one linguistic expression for another expression (definition type 3b), and a definition where a previous definiendum-to-definiens relationship is generally affirmed, but where a *precise* definiens alteration is proposed for pragmatic, technical, or personal reasons (definition type 3c). The symbolic equivalence of the first sentence is a simple abbreviation. With the second sentence, where it is said that '**p** implies **q**' is equivalent to 'either **p** is false, or **q** is true,' this definition is technical, creative, and aims to be 'fruitful.' Mathematical definitions may serve to just 'abbreviate,' or otherwise serve as 'technical formalizations' so as to precisely understand (or stipulate) what was before, a vague concept.[14]

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[14] As an aside, Robinson provides a historical look at 'definition' and provides his own theory of definition. Robinson contrasts definitions that can be understood as being 'nominal' (i.e. about words) to definitions that are 'real' (about things). He concludes that although the concept of 'real definition' has a long history, discussed by Socrates (and Plato), as well as by Aristotle and Spinoza, among others, that this concept should be abandoned. Robinson believes that real definitions, being about things, involve a confusion of twelve activities (p. 189). He advises us that we are better to drop the term 'real definition' and proposes by 'definition' that we "always mean a process concerning symbols, a process either of equating two symbols or of reporting or proposing a meaning for a symbol... " (p. 191). Robinson's hostility against 'real definition' is effectively rebuffed by Copi, Kornblith, and here, with our preferred term 'theoretic definition' taking the place of 'real definition.' Robinson recognizes the existence of 'lexical definitions' which is an assertion that certain people use a certain word in a certain way, and is therefore either true or false. He also recognizes a 'stipulative definition' that has no truth value, and is more like a *request* to the reader to understand the word in a certain way, or a *command*.

Jerry Fodor

The philosopher and cognitive scientist, Jerry Fodor (1935-2017) takes a dim view on the importance of definitions by assuming that there should be an equivalence of meaning between a definiendum and a definiens in order for them to express a synonymy. He states:

> I take the proper ground rule to be that one expression defines another only if the two expressions are synonymous; and I take it to be a necessary condition for their synonymy that whatever the one expression applies to, the other does too. To insist on taking this way isn't, I think, merely persnickety on my part. Unless definitions express semantic equivalences, they can't do the jobs that they are supposed to do… (1998, p. 48).

Fodor misunderstands the nature of definitions. The assertion of a definition is not intended to express a synonymy. A definiendum (as a linguistic entity) has no inherent meaning. Only a definiens is required to have a prior meaning in the specification of any definition. Second, the relation of synonymy is not explainable as just a relation between 'semantic equivalences.' The relation of synonymy is more precisely understood as where a previously defined word may be substituted with another word in any sentential context without change in propositional content. Synonyms are words (or phrases) that are mutually substitutable for another linguistic entity without changing the propositional meaning in any various contexts.[15]

Common Themes in Theories About Definition: Cohen & Nagel

In *An Introduction to Logic and Scientific Method* (1934), Morris Cohen (1880-1947) and Ernest Nagel (1901-1985) devote an entire chapter (Chapter XII. pp. 223-244) to the concept of definition. Their explicit analysis reiterates several common themes concerning the nature of definition.

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[15] As Alan Cruse (2011) notes, the synonymous inter-substitutivity of terms without changing propositional content "is a very severe requirement, and few pairs, if any, qualify... absolute synonyms are vanishingly rare, and do not form a significant feature of natural vocabularies. The usefulness of the notion lies uniquely in its status as a reference point on a putative scale of synonymity" (pp. 142-143). John Saeed (2009) concurs that "exact synonyms are very rare" giving couch/sofa, lawyer/attorney, and large/big as examples (p. 65). Fodor's claim that one expression defines another only if the two expressions are synonymous is clearly mistaken. See chapter seven about the concept of synonymy.

Nominal Definition

Cohen and Nagel begin their chapter by stating that "classifying things really involved, or is a part of, the formation of hypotheses as to the nature of things" (p. 223). They observe that the *vagueness* of ordinary words is one of the primary reasons why technical vocabularies must be constructed in the special sciences. That words can be ambiguous is another problem and they describe a case where they allege that J.S. Mill constructs an argument that illicitly uses two senses of 'desirable' to conclude that 'happiness is desirable.' They state that "much of the best effort of human thought must go, therefore, to delimit the vagueness of words and eliminate their ambiguity" (p. 225).

The authors acknowledge the ancient distinction between 'nominal definitions' and 'real definitions.' They define a 'nominal definition' as follows:

> A *nominal definition* is an agreement or resolution concerning the use of verbal symbols. A new symbol called the *definiendum* is to be used for an already known group of words or symbols (the *definiens*). The definiendum is thus to have no meaning other than the definiens. In the *Principia Mathematica* by Whitehead and Russell a definition is written by putting the definiendum to the left and the definiens to the right with the sign of equality between them and the letters "Df." to the right of the definiens. Thus, implication, symbolized by (the horseshoe symbol), is defined thus: p (the horseshoe symbol) q = ~p v q. Df. Or in words, "p implies q" is equivalent in definition to "not p or q." In algebra the same procedure could be followed. Exponents could be introduced as follows a(squared) = a x a. Df.
>
> A nominal definition, then, is a resolution and not anything true or false-- though of course the assertion that anyone has or has not consistently lived up to this resolution may be true or false. And since that which is neither true nor false cannot be a proposition, nominal definitions cannot be real premises of any argument. There are no implications of truth or falsity in words themselves. But while nominal definitions do not extend our real knowledge, they aid in scientific inquiry… we economize space, time, and attention or mental energy if we use a new and simple symbol for a group of old familiar ones. Thus, if we continued to use ordinary words and did not introduce such technical terms of higher mathematics and physics as "differential coefficient," "energy," "entropy," and the like, our expressions would become so long and involved that we could not readily grasp the complex relations indicated by these terms. Thus, it is easier to read Newton's Principia translated into the technical language of the modern calculus than in the more familiar language of geometry in which Newton wrote. (pp. 228- 229).

Definition by Denotation

Cohen and Nagel continue their chapter in a section entitled "Definition by Denotation" (p. 229). They state that another way that "the meaning of words is clarified is by exhibiting a part of their denotation." What the authors have in mind, is that to understand the meaning of the word, is to give examples of what items fall under a term, as exhibiting a part of their denotation. They illustrate, that when Euthyphro attempted to define 'piety' by giving examples to Socrates, this was unsatisfactory, since that which is offered as an example of piety may also be an instance of something else.

Real Definition

The next topic of their discussion is that of 'real definition.' Continuing with the example of 'piety' in which mere examples constituted just a 'verbal definition' as to what sort of acts the term could be applied correctly; the second attempt by Euthyphro to say what 'piety' is, was in the form of a 'real' definition:

Piety = that which is dear to the gods. Df.

Socrates was searching for an *analysis* of that which the term represented, and this definition is an example of a real definition. The authors continue:

> Like a nominal definition, this real definition defines the word 'piety' by means of an equivalent group of words. But, and this is an important point, the definiens is an analysis of the idea, form, type, or universal symbolized by 'piety.' Both the definiens and definiendum refer to the same thing or character. They each possess a meaning independently of the process of definition which equates them. The definiens, however, indicates the structure of that to which both refer (p. 230).

A Mistaken Example of a Real Definition

The authors at page 230, indicate that some mathematical definitions are a species of 'real definitions' whereby the *definiens* of a mathematical definition (i.e. the right side) offers an analysis of what "both sides symbolize":

> A real definition, therefore, is a genuine proposition, which may be either true or false. Since the definiendum and the definiens much symbolize the same universal, and since the definiens must express the structure of that universal, a real definition can be true only if the two sides of the definition are equivalent in meaning and the right-hand side represents a correct analysis of it.

> We may give another illustration of real definitions. Everyone may be supposed to be familiar with the meaning of 'similar figures.' Such figures resemble one another in a way most people untutored in geometry would find it hard to state, but which they can identify in a crude way. The following is a real definition of *similarity*:
>
> > Figure A is similar to figure A' = The ratio of the distance between any two points P, Q, on A and the distance between the corresponding points P', Q', on A' is constant. Df.
>
> This is a *true definition* of what is ordinarily meant by similar figures, because the right-hand side means precisely what the left-hand side does and at the same time the right-hand side offers an analysis of the structure of that which both sides symbolize (*italics added*, pp. 230-231).

Critically, it seems that the authors are mistaken that this is a "true definition" of what is ordinarily meant by 'similar figures' and that the term can be explicated as a 'real definition' where the definiens now has objective identity conditions. But with anti-realist arguments given in previous chapters, it is maintained here that the structure of 'similar figures' isn't something that exists independent of persons and cannot be subject to a theoretic or real definition. The mathematical definition of 'similar figures' is neither true nor false. It is a fixed definiens concept that is stipulated and adopted by mathematicians for its fruitfulness and consistency.

'Definition': Raziel Abelson- The Encyclopedia of Philosophy

In 1967, *MacMillan* & *The Free Press*, published an ambitious eight-volume set about various philosophical topics and philosopher biographies in a one-time edition. The following portions of text are from *The Encyclopedia of Philosophy* taken from the entry on 'definition.' Its author is Raziel Abelson, a philosopher specializing in metaethics.

What is notable in this entry is that the three kinds of definition hypothesized in the tripartite theory, are mirrored here in a vague way. The tripartite "reportive-theoretic-stipulative" distinction has strong similarities to the "linguistic-essentialist-prescriptive" distinction. Historically, instead of recognizing three kinds of definition, it has been thought that one of these options (as a theory) is better than the other two; but without much extended discussion as to why. We will follow text excerpts from 'definition.'

> The problems of definition are constantly recurring in philosophical discussion, although there is a widespread tendency to assume that they have been solved. Practically every book on logic has a section on definition in which rules are set down and exercises prescribed for applying the rules, as if the problems were all settled. And yet, paradoxically, no problems of knowledge are less settled than those of definition, and no subject is in need of a fresh approach. Definition plays a crucial role in every field of inquiry, yet there are few if any philosophical questions (what sort of thing it is, what standards it should satisfy, what kind of knowledge, if any, it conveys) on which logicians and philosophers agree.
>
> In view of the importance of the topic and the scope of disagreement concerning it, an extensive reexamination is justified. In carrying out this conceptual reexamination, this article will summarize the main views of definition that have been advanced, indicate why none of these views does full justice to its subject, and then attempt to show how the partial insights of each might be combined in a new approach (pp. 314-315).
>
> All of the views of definition that have been proposed can be subsumed under three general types of positions, with needless to say, many different varieties within each type. These three general positions will be called 'essentialist,' 'prescriptive,' and 'linguistic.'
>
> Writers whose accounts of definition are of the 'essentialist' type include Plato, Aristotle, Kant, and Husserl. Supporters of the 'prescriptive' view includes Pascal, Hobbes, Russell, Quine, Goodman, Carnap, and Hempel, as well as most contemporary logicians. Supports of the 'linguistic' view include Mill (in part) and Moore (in part) and Richard Robinson, and most members of the school of linguistic analysis.

We paraphrase as follows (p. 314):

> According to *essentialist* views, definitions convey more exact and certain information than is conveyed by descriptive statements. Such infallible information is acquired by 'intuition,' 'reflection,' or 'conceptual analysis.'
>
> *Prescriptive* views agree with essentialism that definitions are incorrigible, but account for their infallibility by denying that they communicate information and by explaining them as symbolic conventions.
>
> Although *linguistic views* agree with essentialism that definitions communicate information, they also agree with prescriptivism in that they reject claims that definitions communicate information that is indubitable. The linguistic position is that definitions are empirical (and therefore corrigible) reports of linguistic behavior.

The following is paraphrased text:

> (1) Plato distinguished two kinds of objects of knowledge (sensible things and forms) and two modes of knowledge (sense perception and intellectual vision). Definitions describe forms, and since forms are perfect and unchanging, definitions, when arrived at by proper procedure, are precise and rigorously certain truths (p 315).
>
> (2) Aristotle explained the nature of "real" as distinguished from "nominal" (that is, prescriptive, or linguistic) definition (p. 316).
>
> (3) Denying that definitions are statements of any kind, the *prescriptivist* assimilates definitions to imperative sentences rather than to declarative sentences and endows them with the function of syntactic or semantic rules for prescribing linguistic operations. There are two main varieties of prescriptivism. The nominalist variety explains definitions as semantic rules for assigning names to objects, while formalist variety regards definitions as syntactic rules for abbreviating strings of symbols (p. 317).

From a section entitled 'Nominalism':

> For Bacon and Hobbes, definitions possessed a therapeutic function, as a means of clearing up or avoiding ambiguous, vague, and obscure language. Regarding semantic confusion as the main source of intellectual trouble, they proposed to clear the way for a new system of knowledge by subjecting existing concepts to the test of definitional reduction to observable and measurable properties. Definition was thus a surgical knife for cutting away metaphysical encrustations.
>
> Definitions thus clear up ambiguities and "settle significations," rather than communicate information about a realm of essences. According to Hobbes, all knowledge consists in the "right ordering of names in affirmation." A proposition connects one name to another, and an inference adds or subtracts one proposition from another. The structure of scientific thought thus maps the structure of the physical world. It would seem then that, for Hobbes, all scientific knowledge is derivable from definitions. Yet Hobbes also stressed the role of perception in knowledge.
>
> The solution to this paradox lies in Hobbes' conception of naming. All inquiry is deductive except for the assignment of names to things, and it is to the assignment of names that we must look for the empirical sources of knowledge. But if follows that definitions as assignments of names must be as informative for Hobbes as they are for Plato or Aristotle. This conclusion leads to a further paradox, for, according to Hobbes, definitions provide no information at all; they express conventional decisions to use particular signs as names of particular objects (p. 318).

From a section entitled 'modern formalism' (p. 320) Abelson's entry follows:

> Rudolf Carnap and C.G. Hempel have tried to clarify the difference between informative definitions and mere notational abbreviations by distinguishing between "old" and "new" concepts. Definitions of old concepts are called "explications" by Carnap and "rational reconstructions" according to Hempel, while both call definitions of new concepts "notational conventions." When we are "explicating" or "reconstructing" a concept, our definitions are subject to evaluation by the criteria of the conformity to usage and increase precision (Carnap, 1937, *The Logical Syntax of Language*, p. 23).
>
> When definitions are introduced solely for the purpose of abbreviation, only the criterion of consistency applies. One must therefore wonder why Carnap and Hempel should bother to call notational abbreviations "definitions," since they have nothing whatever in common with explications.
>
> Perhaps the answer to this question lies in the logical difficulties lurking within the notion of explication. What does it mean to "reconstruct" or "explicate" a concept, and what precisely is the difference between "old" and "new" concepts? If definitions of old concepts must conform to established usage, are they not true or false statements about language usage, in which case the distinction between definitions and empirical statements disappears?

From a section entitled 'linguistic theories':

> The clearest formulation of the *linguistic* view was provided by Richard Robinson in his book Definition, which has the distinction of being the only book in the English language devoted to this subject (p. 320).
>
> Robinson formulated a purely linguistic account of definitions as reports of word usage. But he thought it necessary to supplement his main view with a "stipulative," or prescriptive account.
>
> The reasons for his vacillation are that reports of usage are empirical generalizations, while definitions are, if acceptable at all, necessary truths, and that stipulations are uninformative, while definitions are highly informative. Thus, neither the linguistic nor the prescriptive interpretation accounts for all features of definitions. But the mere juxtaposition of the two can hardly overcome the defects of each taken separately (p. 321).

From a section entitled 'rules for defining' (p. 322):

> A number of rules of thumb for evaluating definitions have become canonical in the literature on the subject despite the fact that they make no clear sense in terms of any of the traditional views. The following can be found in practically every textbook on logic. They were first suggested by Aristotle in his *Topica* and have survived without change by sheer weight of tradition:
>
> (1) A definition should give the essence or nature of the thing defined, rather than its accidental properties.
>
> (2) A definition should give the genus and differentia of the thing defined.
>
> (3) One should not define by synonyms.
>
> (4) A definition should be concise.
>
> (5) One should not define by metaphors.
>
> (6) One should not define by negative terms or by correlative terms (e.g., one should define north as opposite of south, or parent as a person with one or more children).

Abelson finds problems with the consistency and applicability of these rules.[16] He then closes out his article with a "pragmatic-contextual" approach to definition, which we won't discuss here.

Conclusion

In sum, this appendix supports a view that the definitions used by mathematicians are typically of the 3b abbreviation type of definition, and the 3c precise formalization type of definition (including recursive definitions).[17]

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[16] Linda Zagzebski (1999) recites similar criteria for a good definition (p. 98): 1) It shouldn't be *ad hoc*. 2) It shouldn't be negative when it can be positive. 3) It should be brief. 4) It shouldn't be circular. 5) It should use concepts that are less obscure than the concept (term) to be defined. 6) A definition is supposed to tell us something we didn't know.

[17] Another interesting and detailed history of 'definition' is that of David Wiggins (2007) "Three Moments in the Theory of Definition or Analysis: Its Possibility, Its Aim or Aims, And Its Limit or Terminus" in *Proceedings of the Aristotelian Society. Vol, CVII, Part 1.*

References

Abelson, Raziel 1967. 'Definition.' *The Encyclopedia of Philosophy*. New York: MacMillan Publishing Co. and The Free Press.

Ayer, A.J. 1946. *Language, Truth and Logic* (2nd edition). New York: Dover Publications, Inc.

Brown, James Robert 2008. *Philosophy of Mathematics* (2nd edition). New York: Routledge.

Carnap, Rudolf 1937. *The Logical Syntax of Language*. London: Routledge and Kegan Paul.

Cohen, Morris R. and Nagel, Ernest 1934. *An Introduction to Logic and Scientific Method*. New York: Harcourt, Brace and Company.

Cruse, Alan 2011. *Meaning in Language: An Introduction to Semantics and Pragmatics*. New York: Oxford University Press.

Dummett, Michael 1981. *The Interpretation of Frege's Philosophy*. Cambridge, MA: Harvard University Press.

Fodor, Jerry A. 1998. *Concepts: Where Cognitive Science Went Wrong*. New York: Clarendon Press.

Frege, Gottlob 1879. *Begriffshrift* in Peter Geach and Max Black (eds) 1970. *Translations from the Philosophical Writings of Gottlob Frege.* Oxford: Basil Blackwell.

Heath, Thomas L. (ed.) (1925, 1956). *Euclid: The Thirteen Books of the Elements* (2nd ed.). New York: Dover Publications, Inc.

Hodges, Wilfrid 2008. 'Tarski's Theory of Definition' in Douglas Patterson (ed.) *New Essays on Tarski and Philosophy*. New York: Oxford University Press.

Horty, John 2007. *Frege on Definitions.* New York: Oxford University Press).

Kornblith, Hilary 1993. *Inductive Inference and Its Natural Ground.* Cambridge, MA: The MIT Press.

Mill, John Stuart 1875. 'Of Names.' From *System of Logic*, Book 1, Chapter 2, Section 5. Reprinted in Martinich 2001.

Mill, John Stuart 1882. *A System of Logic, Ratiocinative and Inductive* (8th edition). Project Gutenberg E-Book 27942 (PDF, online).

Popper, Karl R. 1945. 'Two Kinds of Definition' *The Open Society and Its Enemies* (5th edition). Princeton, NJ: Princeton University Press.

Quine, W.V.O. 1936. 'Truth by Convention' in Otis H. Lee (ed.) *Philosophical Essays for A.N. Whitehead.* New York: Longmans, Green & Co.

Quine, W.V.O. 1953. *From a Logical Point of View*. Cambridge, MA: Harvard University Press.

Resnik, Michael D. 1980. *Frege and the Philosophy of Mathematics*. Ithaca, NY: Cornell University Press.

Robinson, Richard 1954. *Definition*. London: Oxford University Press.

Ross, G. MacDonald 1984. *Leibniz*. New York: Oxford University Press.

Russell, Bertrand and Whitehead, Alfred 1903. *Principia Mathematica*. Cambridge, UK: Cambridge University Press.

Saeed, John I. 2009. *Semantics* (Third edition). Malden, MA: Blackwell Publishing.

Schlick, Moritz 1925, 1985. *General Theory of Knowledge*. LaSalle, IL: Open Court Publishing.

Suppes, Patrick 1957. *Introduction to Logic*. Mineola, NY: Dover Publications, Inc.

Tarski, Alfred 1946. *Introduction to Logic and the Methodology of Deductive Sciences*. New York: Oxford University Press.

Venema, Gerard A. 2002. *The Foundations of Geometry*. Upper Saddle River, NJ: Pearson Prentice-Hall.

Wiggins, David 2007. 'Three Moments in the Theory of Definition or Analysis: Its Possibility, Its Aims or Aims, and Its Limits or Terminus.' *Proceedings of the Aristotelian Society* 107: 73-109.

Zagzebski, Linda 1999. 'What is Knowledge?' in Greco and Sosa (eds.). *The Blackwell Guide to Epistemology*. Malden, MA: Blackwell Publishing.